Hutton All Saints' C. of E. Primary School (V.A.)

Commitment to Excellence within a Caring, Christian Environment.

Calculation Policy

Date of Policy:	**November 2013**
Reviewed:	
Member(s) of staff Responsible:	**Miss Dowson**
Review Date:	**November 2015**

Rationale and Aims

This policy is intended to aid all staff in understanding the four rules and help provide progression in written calculations. It builds upon the 2014 curriculum for maths and should be used in conjunction with the school's mathematics policy. The policy recognises that pupils' mental number knowledge and skills are of prime importance. The content of this policy is to give guidance regarding progression which the majority of pupils will make.

It is expected that addition/subtraction and multiplication/division be taught alongside each other so that pupils can see and use the relationship between them. Children should also be encouraged to recognise multiplication as repeated addition and division as repeated subtraction. Pupils should be taught to estimate their answers first and check calculations with a variety of strategies including the inverse operation.

When approaching a calculation, children should be encouraged to ask themselves the following questions…

- → Can I do this in my head?
- → Do I know the approximate size of the answer?
- → Do I need a calculator to work this out?
- → If I can't answer it wholly in my head, what do I need to write down in order to help me calculate the answer? (jottings)
- → Which written method would be helpful?
- → Which method would be most efficient?

Wherever appropriate, children should do a mental calculation. For example, which of these would you do mentally?

3002 – 2998 9563 – 3771

Steps towards adding and subtracting mentally any pair of two-digit numbers

Step 1:

U +/- 1		3 + 1	4 - 1
U +/- U	Not crossing the tens boundary	2 + 4	6 - 4
10 +/- U		10 + 4	10 - 4
TU +/- U	Not crossing the tens boundary	12 + 4	16 – 4

Step 2:

T +/- U		50 + 4	50 - 4
TU +/- 10	Not crossing the hundreds boundary	52 + 10	62 - 10
T +/- T	Not crossing the hundreds boundary	50 + 30	80 - 30
TU +/- U	Not crossing the tens boundary	52 + 4	56 - 4
U + U	Crossing the tens boundary	6 + 8	
TU +/- U	Crossing the tens boundary	15 + 8	15 - 8
TU +/- T	Not crossing the hundreds boundary	52 + 30	82 - 30
TU +/- U	Crossing the tens boundary	55 + 8	63 - 8

Step 3:

TU +/- TU	Not crossing the tens or hundreds boundary	52 + 14	66 – 14
TU +/- T	Crossing the hundreds boundary	92 + 10	102 - 10
T +/- T	Crossing the hundreds boundary	80 + 50	130 – 80

Step 4:

T +/- TU	Crossing the tens boundary or the hundreds boundary	80 + 52	80 - 52
TU +/- TU	Crossing the tens boundary but not the hundreds boundary	55 + 18	73 - 18
TU +/- TU	Not crossing the tens boundary but crossing the hundreds boundary	52 + 84	136 - 84
TU +/- TU	Crossing the tens boundary but also the hundreds boundary	55 + 78	133 - 78

Mental methods should be secure and understanding of place value **before** children are adding and subtracting using written methods. This will mean that they are able to apply their mental calculations to their written algorithms.

E.g.
Before solving 43 +52 using a written method, children should be secure in their understanding of place value in two-digit numbers and be confident adding multiples of 10 and units, mentally.
Moving onto solving 100 – 49, children should already be confident in knowing their 10s number bonds for 100 and their number bonds to 10.

Addition

Stage 1

Counting, knowing the order of numbers, lots of practical activities with no written recording.

Combine two groups of objects and begin to record pictorially.

E.g. Jane has 3 bears. She was given 2 more. How many does she have now?

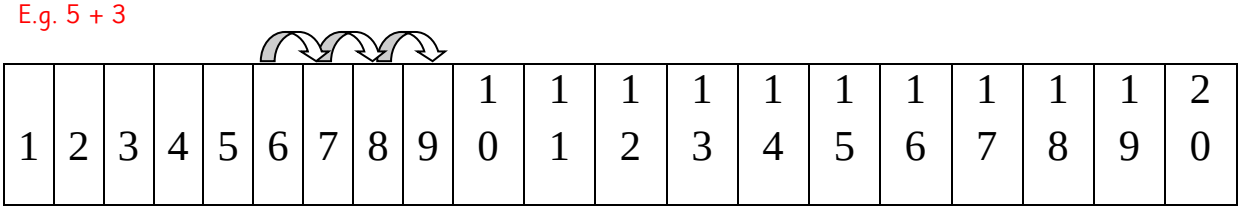
3 2 3 + 2 = 5

Children will use a mixture of **words and symbols** in order to explain to someone else the methods they have used.

Stage 2

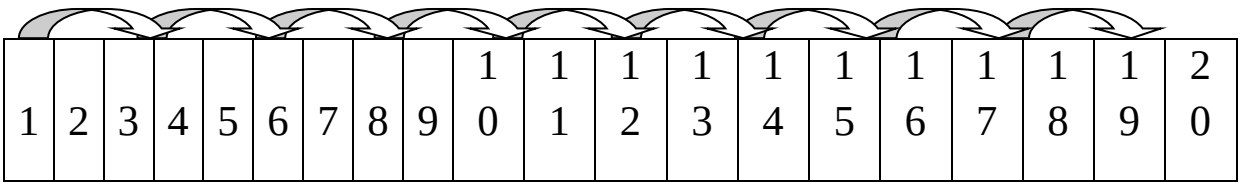
The use of **numbered number tracks and lines** is very helpful for teaching children the order of numbers and for images of addition and subtraction. It may begin with children physically jumping forwards and backwards along a number track. Number lines should be available in all EYFS and Year 1 classrooms for children to use independently.

E.g. 5 + 3



Children can then use the track for finding patterns

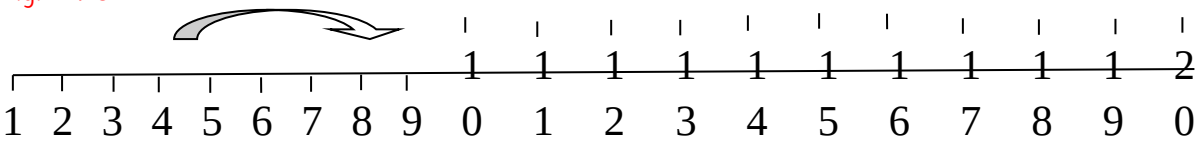
E.g. Mark the numbers you land on when you hop forward in twos from different starting numbers



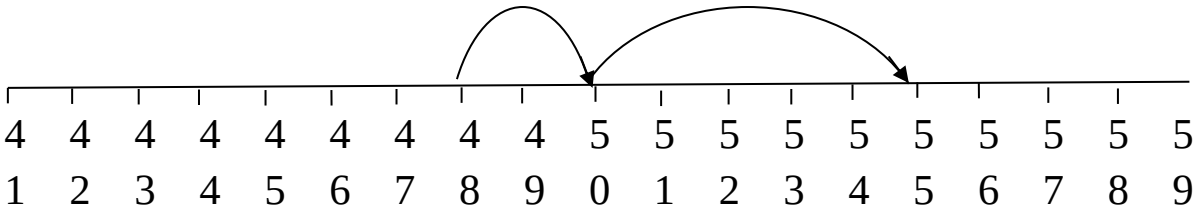
Stage 3

Building on mental methods using a **completed number line**

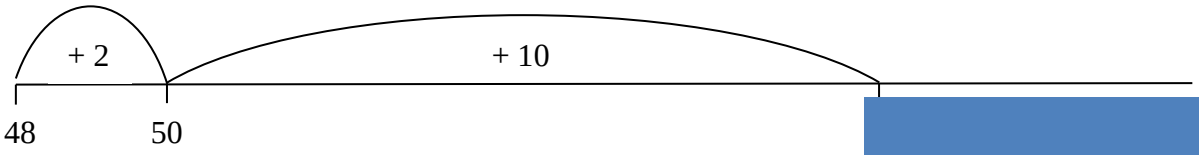
E.g. 4 + 3 = 7



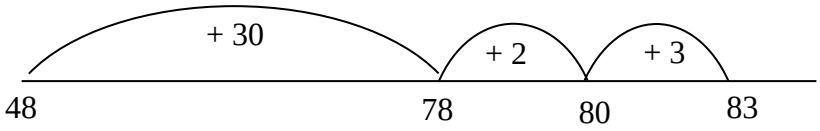
E.g. 48 + 7 = 55 (Bridging through tens boundary 8 +2 +5)



E.g. 48 + 12 = 60 (Bridging through tens boundary on an empty number line 48 + 10 + 2)



E.g. 48 + 35 = 83



Possible Success Criteria
- ✓ Start at the largest number
- ✓ Jump in tens
- ✓ Partition remaining units to bridge through ten
- ✓ Check and change by adding up jumps to check they are the number you are adding.
- ✓ Use a hundred square to support them.

The empty number line helps to record the steps on the way to calculating the total, those children who are less secure adding thirty may record three jumps of 10.

Pupils will be encouraged to start with the largest number, this may require them to reorder the calculation. When calculating, pupils build on mental methods by using number bonds, then jumping in tens and partitioning remaining units to bridge through ten. Always encourage children to check their results thoroughly, recalculating each step.

At this point, partitioning numbers into tens and units and adding them separately would also be introduced as a more efficient method. This will lead towards when numbers are too big to hold in our heads to calculate. Children learn to partition the numbers into tens and units, add these together and then add these numbers together.

E.g.

```
4  7  +  7  6  =
4  0  +  7  0  =     1  1  0
   7  +     6  =        1  3
1  1  0  +  1  3  =  1  2  3
```

```
1  4  7  +  4  3  2
1  0  0  +  4  0  0  =  5  0  0
   4  0  +     3  0  =     7  0
         7  +        2  =        9
5  0  0  +  7  0  +  9  =  5  7  9
```

Further example of why place value understanding and mental strategies need to be secure before written method.

Stage 4

This can also be represented vertically and we call this method the **expanded column method**. This step is an important layout as it helps children to understand what is happening when a number is carried in the vertical algorithm. This method can be used to introduce the column method and explain what is happening as we 'carry' numbers to the next column.

Using cuisenaire tens rods and units cubes are recommended to reinforce understanding of place value in this method.

Possible Success Criteria
- ✓ Write the numbers in the correct columns
- ✓ First add the units
- ✓ Then add the tens
- ✓ Then add the hundreds
- ✓ Add your new set of numbers
- ✓ Check each of your calculations is right.

Examples:

```
  47
+ 76
  13
 110
 123
```

```
  368
+ 493
  11
 150
 700
 861
```

Children can have HTU written above the relevant columns. Teaching children to have one number in each square is particularly important here.

Stage 5

Before year six, every child should be confident in calculating using the **compact column method**. In this stage, the recording is reduced further:

Examples:

```
  47
+ 76
 123
 11
```

```
  368
+ 493
  861
  11
```

```
  864
+ 383
 1247
 11
```

Addition using decimal numbers

In Year 2 children will be introduced to decimal notation in the form of mixed money values. Around year three, children will be taught that decimal notation allows us to record tenths of a number with an integer. Children should calculate the sum of decimal numbers using the same method they use for addition of integers.

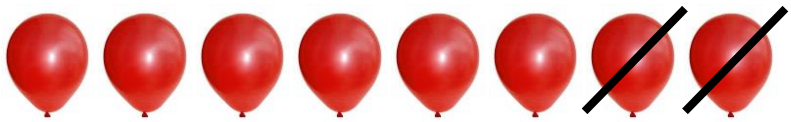
Subtraction

Stage 1

Counting backwards, knowing the order of numbers, lots of practical activities with no written recording.

Children need practical activities of taking away by finding how many are left from a collection of objects when some are removed.

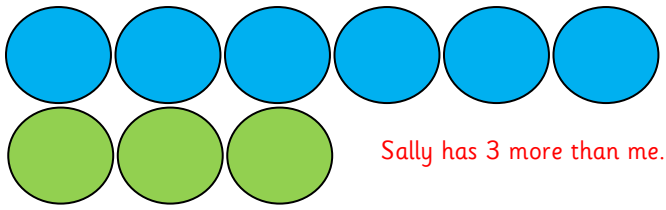
E.g. There were eight balloons. Two popped. How many are left?



8 – 2 = 6

Children also need practical activities around 'finding the difference', which involves making a comparison between numbers in two groups. They need to recognise that this is another form of subtraction.

E.g. How many more biscuits does Sally have than you? (biscuits represented by counters)



Sally has 3 more than me.

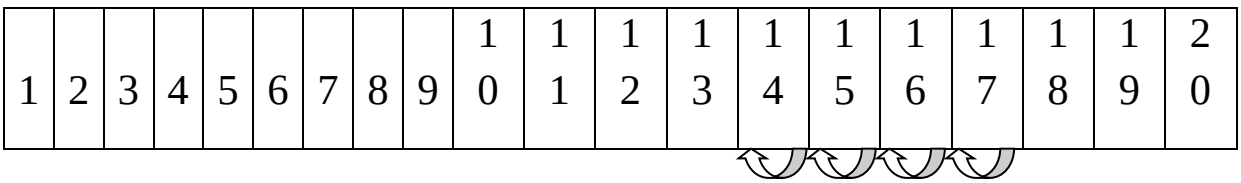
A mixture of **words and symbols** will be used by children in order to explain to someone else the methods that they have used. Children will use a variety of ways of recording subtraction, reflecting the mental methods they have used.

Possible Success Criteria – finding the difference

- ✓ Numbers the correct way round on the number line.
- ✓ Jump in tens
- ✓ Add any units to bridge through the tens boundary.
- ✓ Check you are at the big number.
- ✓ ADD UP YOUR JUMPS
- ✓ Write the answer in the number sentence
- ✓ Check your jumps and change if

Stage 2

Again, the use of **numbered number tracks and lines** is very helpful for teaching children the order of numbers and for images of addition and subtraction. When recording subtraction on a number track or line children should draw arrows underneath the line to reinforce the inverse relationship with addition.



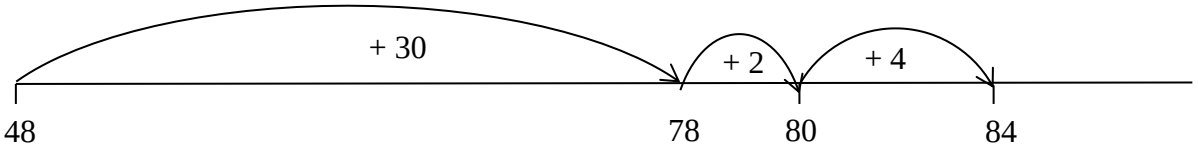
NB: Children will be taught to calculate both by counting back and by counting on to 'find the difference', depending on the size of the numbers.

Stage 3

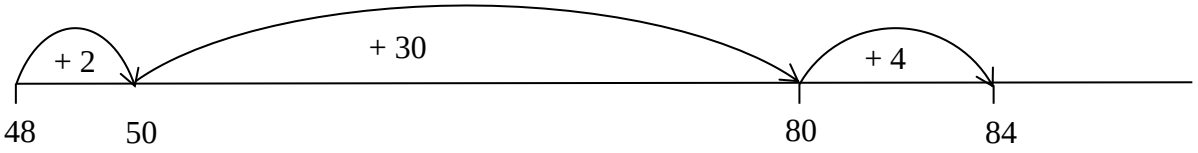
Using **blank number lines** to 'find the difference'

E.g. 81 - 79 = 2 (As a finding the difference or counting on)

E.g. 84 - 48 = 36 (As a finding the difference or counting on)

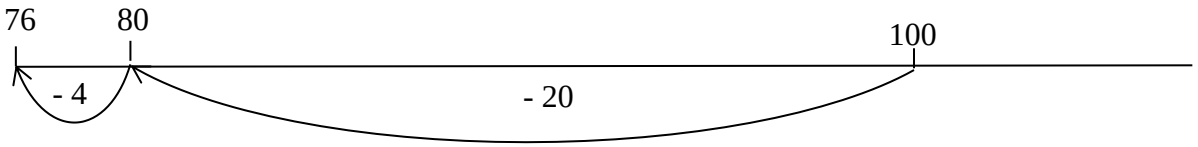


E.g. 84 - 48 = 36 (As a finding the difference or counting on)



Using **blank number lines** to 'jump backwards'.

E.g. 100 - 24 = 76(By counting backwards)



Stage 4 **PARTIONING**

Like the final step for addition, pupils now progress to using the **expanded layout** which leads to the column method. Using this interim stage, again, helps children to understand the column method, particularly when decomposition is required.

Numbers should be partitioned and written one under the other which mirrors the column method where tens are placed under tens and units under units. This is a stage which aids mental calculation and is probably only worth teaching to the more able mathematicians who will cope with recognising that this method can only be used when the units digit is larger in the first number.

E.g. 74 – 23 =

4 – 3 = 1
70 – 20 = 50
51

This can be applied to 3 or 4 digit numbers.

E.g. 384 – 173 =

This method leads to a more **compact column method**, firstly without decomposition and then leading onto 'exchanging' digits, as below. Please do not use the vocabulary 'borrow' as the exchanged digit does not get given back.

E.g. 73 – 24 =

```
  6  1
  7  3
- 2  4
  4  9
```

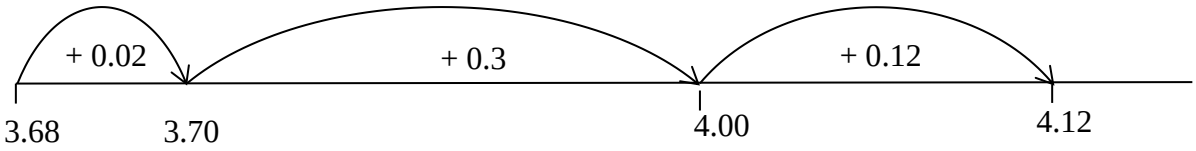
which can be applied to 3 or 4 digit numbers →

563 – 271

```
  4  1
  5  6  3
- 2  7  1
  2  9  2
```

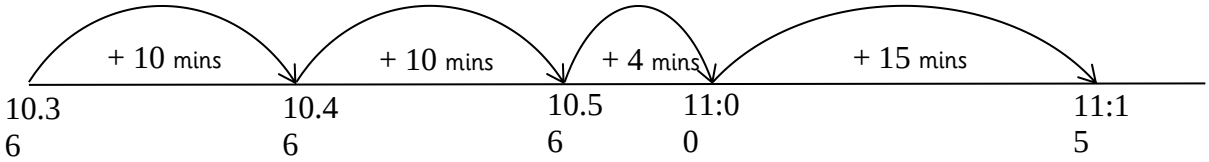
The empty number line is useful to use at any stage of development, particularly when introducing a new number concept to ensure that children understand the place value of each digit.

E.g. Decimals : 4.12 – 3.68



Mentally: 0.02 + 0.3 + 0.12 = 0.44

E.g. Elapsed time : Find the difference between 10:36 and 11:15



Mentally: 10 + 10 + 15 + 4 = 39 mins

Should children add minutes and it equal more than an hour e.g. 80 mins, they should ensure they convert this back into hours and minutes.

Subtraction using decimal numbers

As with addition, Year 2 children will be introduced to decimal notation in the form of mixed money values. Around year three, children will be taught that decimal notation allows us to record tenths of a number with an integer. Children should calculate the difference between decimal numbers using the same method they use for subtraction of integers.

Multiplication

Stage 1

It is expected that there will be lots of practical activities to support children's growing awareness and understanding of multiplication. Children can complete practical activities involving grouping objects. Rhymes and stories can be used that involve counting in number. Apparatus should be used to sort objects into groups.

E.g. Ice-creams come with 2 scoops. How many scoops do these three ice-creams have?

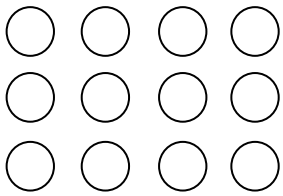
3 lots of 2 3 groups of 2

2 + 2 + 2 3 x 2 (say 3 multiplied by 2)

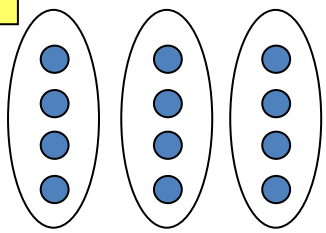
A mixture of **pictures, words and symbols** will be used by children in order to explain to someone the methods that they have used.

ARRAY

1. Draw 3 circles
2. Draw 4 dots in each one
3. Count the total number of dots.



3 x 4 can be shown as above with an array.

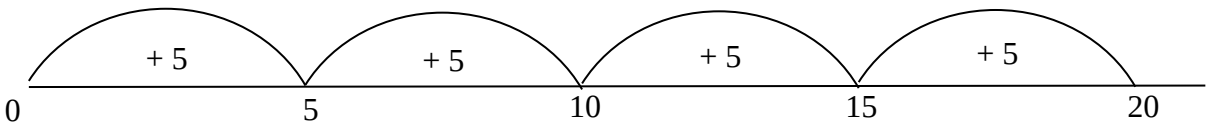


Which the children may then represent as above.

Stage 2

Children will begin to recognise multiplication as repeat addition.

E.g. What is the value of 4 five-pence coins?



To be able to use written methods of multiplication successfully, it is important that children's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for multiplication facts. For example:

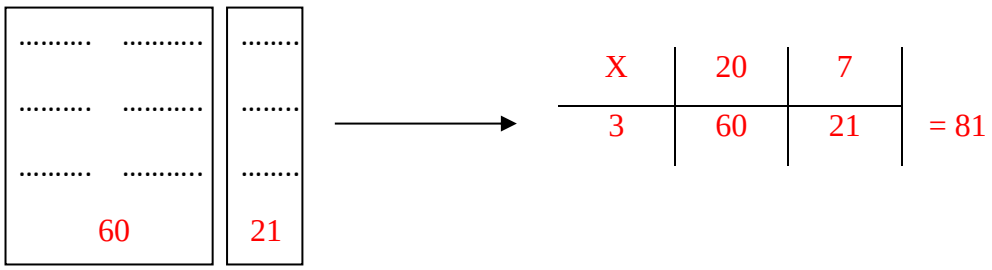
- ✓ Recalling multiplication facts
- ✓ Multiplying by 10 and 100
- ✓ Partitioning numbers into multiples of hundreds, tens and units
- ✓ Add two or more single digit numbers mentally
- ✓ Add multiples of 10 or 100
- ✓ Add combinations of whole numbers using the column method

Stage 3a

TU x U

Link **arrays** to the **grid method** and then use the grid method, as shown below, to solve number sentence. The grid method reinforces the commutative law in multiplication.

E.g. 27 X 3



Stage 3b

TU x U

This step makes the link between the grid method and the vertical format. The two formats can be used alongside each other once children are secure with the grid method. Introducing this expanded format before the traditional column method helps children to understand the column method and therefore apply it more effectively.

E.g. 27 X 3

```
  2 7
X   3
-----
  2 1   (7 X 3 )
  6 0   (2 0 X 3 )
-----
  8 1
```

Possible Success criteria:

- ✓ Draw your grid and partition the numbers
- ✓ Multiply your units
- ✓ Multiply your tens
- ✓ Add the answers together
- ✓ Check each of your calculations and change if you need to.

Stage 3b

TU x U

This final step involves children using the vertical format with the compact working . When pupils begin to attempt 'long multiplication number sentences' it may be useful for them to make the link with this and the grid method and for pupils to use informal jotting to support their calculations.

```
  6 7
X   3
-------
2 0 1
-------
  2
```

The language tens and units should be used repeatedly when teaching this method.
E.g. Next I multiply these six tens or sixty by three – how much will I have?

Children should be secure in the methods 3a and 3b before using the compact column method so that they understand the process of this method.

Stage 4a

TU x TU

As this will be a new concept, children should go back to the simplest layout – the grid method.

E.g. 27 X 56

X	20	7
50	1000	350
6	120	42

→

1	0	0	0
	3	5	0
	1	2	0
+		4	2
1	5	1	2

If children are confident with their mental addition, this second step can be done mentally.

Stage 4b

TU x TU

As with the TU x U, children can learn the expanded column method alongside the grid method, once the grid method is secure. This interim step will help children to understand the process of the traditional column method.

		2	7
	X	5	6
		4	2
	1	2	0
	3	5	0
1	0	0	0
1	5	1	2

6 X 7
6 X 20
50 X 7
50 X 20

If children are using this method they should write these number sentences but should check that they have calculated each of these steps.

Stage 4c

TU x TU

		2	7
	X	5	6
	1	6_{4}	2
1	3_{2}	5	0
1	5	1	2

When calculating with these written recordings, children should be able to recognise that they are still multiplying 5 tens by 7. This way they do not need to 'add a 0' the answer is 50.

Stage 5a

HTU x TU

Again, children should go back to the grid method when learning a new level of place value to multiply:

E.g. 286 X 29

X	200	80	6	
20	4000	1600	120	= 5720
9	1800	720	54	= 2574

→

$$\begin{array}{r} 5\ 7\ 2\ 0 \\ +\ 2\ 5\ 7\ 4 \\ \hline 8_1\ 2\ 9\ 4 \end{array}$$

Stage 5b

HTU x TU

Stage 5c

HTU x TU

This can be used as a teaching step to demonstrate the compact column method.

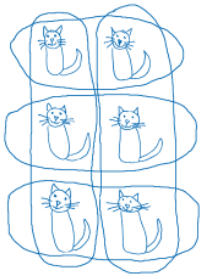
$$\begin{array}{r} 2\ 8\ 6 \\ X\quad 2\ 9 \\ \hline 5\ 4 \\ 7\ 2\ 0 \\ 1\ 8\ 0\ 0 \\ 1\ 2\ 0 \\ 1\ 6\ 0\ 0 \\ 4\ 0\ 0\ 0 \\ \hline 8\ 2\ 9\ 4 \\ \hline 2 \end{array}$$

→

$$\begin{array}{r} 2\ 8\ 6 \\ X\quad 2\ 9 \\ \hline 2\ 5\ 7\ 4 \\ {\scriptstyle 2\ \ 7\ \ 5} \\ 5\ 7\ 2\ 0 \\ {\scriptstyle 1\ \ 1} \\ \hline 8\ 2\ 9\ 4 \\ \hline 1 \end{array}$$

Division

The method developed in this policy is the chunking method. The method in Key Stage One builds on the children's use of number lines with repeat addition.



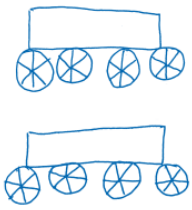
Stage 1

Young children will be familiar with the language of sharing and understand that six shared equally among three people means that everyone has two each and that if they were shared between two people, both would have three.

Stage 2

Children can draw pictures to explain to someone else how they have solved a simple division problem.

E.g. How many cars with four wheels can you make with eight wheels?



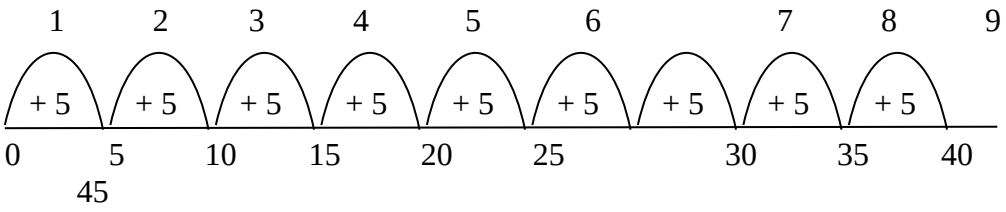
Possible Success criteria: Division with a number line

- ✓ Draw a number line
- ✓ Start at 0
- ✓ Jump in the amount you are dividing by (e.g.5)
- ✓ Count your jumps – this is the answer
- ✓ Write your answer in the number sentence
- ✓ Check your answer

Stage 3

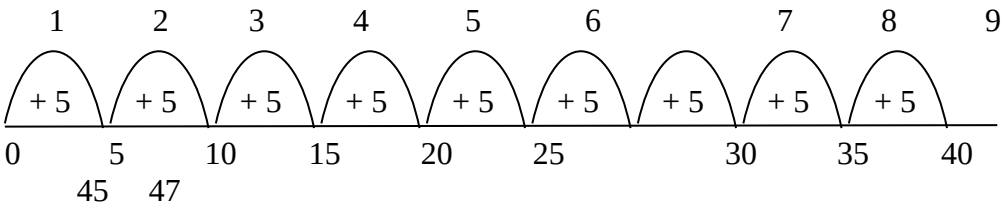
Begin with numbers that children can divide mentally. Develop to using a number line to count on **mentally**. Begin with numbers with no remainders. Number lines do not need to be drawn at this stage.

E.g. 45 ÷ 5 = 9



Then develop to use with remainders, adjusting the above suggested criteria.

E.g. 47 ÷ 5 = 9 r 2



2 left over

This can also be shown with repeat subtraction so that children start to recognise division as repeat subtraction.

Stage 4

Repeated subtraction on a number line can be used with larger numbers by taking away more than one group at a time. Chunking in groups of ten helps.

E.g. 54 ÷ 3 =

For chunking, children need to use times-tables facts to solve division problems.

E.g. 128 ÷ 4

	1 2 8		
-	4 0	1 0	X 4
	8 8		
-	4 0	1 0	X 4
	4 8		
-	4 0	1 0	X 4
	8		
-	8	2	X 4
	0		

Mentally: 10 + 10 + 10 + 2 lots of 4 = 32 lots of 4

This method can also be used when there is a remainder:

E.g. 93 ÷ 3 = 32 r 1

	9 7		
	6 0	2 0	X 3
	3 7		
	3 0	1 0	X 3
	7		
	6	2	X 3
	1		

Children should become used to using quick multiplication facts as below without writing them down:

3 X 2 = 6
3 X 4 = 12 (double 6)
3 X 8 = 24 (double 12)
3 X 10 = 30
3 X 5 = 15 (half 30)
3 X 10 = 60 (double 10)

Children should also recognise from around year 3 that division number sentences may be written as shown here: 3⟌65 They should also know that when written like this they need to divide 65 by 3. They can use chunking to find the answer.

Stage 6

In readiness for year six, children will be introduced to standard written methods of long division:

```
        2 3 r 8
   ____________
2 4 | 5 6 0
    - 4 8 0      20 X 24
      ------
        8 0
    -   7 2        3 X 24
      ------
          8
```

Note – this is the chunking method but with a different layout of the number sentence.

The traditional 'bus stop' method is only used when children are fully secure with chunking. This method should be first introduced with short division (a) and then explained with long division (b).

(a)

```
      1 1 4
   ____________
4 | 4 5 1 6
```

'The digit 1 represents one ten has been exchanged for 10 units.'

(b)

```
        3 2
   ____________
1 6 | 5 5 1 3 2
```